THERMODYNAMIC FUNCTIONS FOR SOME PARTICULAR FORMS OF THE EXPRESSION FOR INTERNAL ENERGY OF AN IDEAL, COMPRESSIBLE MEDIUM

(TERMODINAMICHESKIE FUNKTSII DLIA NEKOTORYKH KONKRETNYKH VIDOV VNUTRENNEI ENERGII IDEAL'NOI ZHIMAEMOI SREDY)

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N.N. KOCHINA
(Moscow)

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A general solution is obtained of the partial differential equation which determines the dependence of temperature on internal energy for an ideal two-parameter medium; the solution is given for several particular forms of the expression for internal energy. In every case considered, the internal energy depends on two arbitrary functions, each of one argument.

Suppose that the internal energy of the medium is given in terms of the pressure p and the density ρ:

$$\varepsilon(p, \rho) = \frac{p_0}{\rho_0} E(P, R) + \text{const} \qquad \left(P = \frac{p}{p_0}, \ R = \frac{\rho}{\rho_0}\right) \tag{1}$$

where p_0 and ρ_0 denote constants, their dimensions being those of pressure and density, respectively. From the fact that

$$d\sigma = \frac{dE + PdR^{-1}}{\tau} \qquad \left(\tau = \frac{T}{T_0}, \ \sigma = \frac{\rho_0 T_0}{p_0} S\right) \tag{2}$$

is a perfect differential it follows that the temperature τ must satisfy the linear partial differential equation [1]

$$\tau + \left(R^2 \frac{\partial E}{\partial R} - P\right) \frac{\partial \tau}{\partial P} - R^2 \frac{\partial E}{\partial P} \frac{\partial \tau}{\partial R} = 0 \tag{3}$$

We write the characteristic equations of (3):

$$\frac{d\tau}{\tau} = \frac{dP}{P - R^2 \partial E / \partial R} = \frac{dR}{R^2 \partial E / \partial P} \tag{4}$$

If the function $E(P, R)$ is of the form

$$E(P, R) = P\varphi(R) \tag{5}$$

where $\phi(R)$ denotes an arbitrary function, it follows that the motion resulting from a strong point-explosion will be self-similar [2]. In this case the system (4) can be integrated by quadratures and the solution to Equation (3) can be written [1]

$$\tau = \exp \int \frac{dR}{R^2\varphi(R)} \Phi(\psi), \qquad \psi = P\varphi(R) \exp\left[-\int \frac{dR}{R^2\varphi(R)}\right] \tag{6}$$

where Φ denotes an arbitrary function. The first of Equations (6) and Equation (2) lead to the following expression for entropy:

$$\sigma = \sigma_0 + \int \frac{d\psi}{\Phi(\psi)} \tag{7}$$

The solution of the problem of the strong point-explosion, subject to the internal energy being of form (5) for three particular forms of $\phi(R)$, has been given in [3].

In [4] we considered the problem of a point-explosion in a compressible medium in the linearized case. Assuming that the internal energy of the medium is given by

$$E(P, R) = P\varphi(R) + \Delta(P, R) \tag{8}$$

where $\phi(R)$ and $\Delta(P, R)$ denote arbitrary functions and where the function $\Delta(P, R)$ is small compared with $P\phi(R)$, it is possible to derive the following expressions for temperature and entropy:

$$\tau = \exp \int \frac{dR}{R^2\varphi(R)} \left\{\Phi(\psi)\left[1 \quad \int \frac{1}{R^2\varphi(R)} \exp\left(-\int \frac{dR}{R^2\varphi(R)}\right) \frac{\partial\Delta(R, \psi)}{\partial\psi} dR\right] + \right.$$

$$\left. + \Phi'(\psi) \int \exp\left(-\int \frac{dR}{R^2\varphi(R)}\right) \frac{\partial\Delta(R, \psi)}{\partial R} dR\right\} \tag{9}$$

$$\sigma = \sigma_0 + \int \frac{1}{\Phi(\psi)} \left[d\psi + \exp\left(-\int \frac{dR}{R^2\varphi(R)}\right) \frac{\partial\Delta(R, \psi)}{\partial R} dR\right] \tag{10}$$

where ψ was given in (6). Without the stipulation that $\Delta(P, R)$ is small these equations represent the general solution of Equation (3) and determine the entropy in accordance with (2), if

$$E(P, R) = P\varphi(R) + \Delta(R) \tag{11}$$

or

$$E(P, R) = P\varphi(R) + \Delta\left[P\varphi(R) \exp\left(-\int \frac{dR}{R^2\varphi(R)}\right)\right] \tag{12}$$

In the case when the function $E(P, R)$ is given by Equations (11) and (12) the relations in (9) and (10) transform to

$$\tau = \exp\int \frac{dR}{R^2\varphi(R)}\,\Phi(\psi), \qquad \sigma = \sigma_0 + \int \frac{d\psi}{\Phi(\psi)} \tag{13}$$

$$\psi = P\varphi(R)\exp\left(-\int \frac{dR}{R^2\varphi(R)}\right) + \int \Delta'(R)\exp\left(-\int \frac{dR}{R^2\varphi(R)}\right)dR$$

$$\tau = \left[\exp\int \frac{dR}{R^2\varphi(R)} + \Delta'(\psi)\right]\Phi(\psi), \qquad \sigma = \sigma_0 + \int \frac{d\psi}{\Phi(\psi)} \tag{14}$$

$$\psi = P\varphi(R)\exp\left(-\int \frac{dR}{R^2\varphi(R)}\right)$$

Equations (13) have also been obtained by V.P. Korobeinikov, and Equations (14) by Iu.L. Iakimov (in 1959 in a thesis on the propagation of shock waves in ideal media of arbitrary physical properties), who proved that the equation of motion (without taking into account the conditions on the shock wave) is self-similar in the case of a medium whose internal energy is of the form (12). Reference [5] treats the problem of a point-explosion in a medium whose internal energy is given by (12) in the special case where $\phi(R) = 1/6\ R$.

It turns out that in addition to the above two cases Equation (4) can be solved by quadratures and that it is possible to find a general solution of Equation (3) for media whose internal energy is given by

$$E(P, R) = \frac{1}{R}\varphi(P) + \Delta(P) \tag{15}$$

where

$$\tau = \exp\int \frac{dP}{[P + \varphi(P)]}\,\Phi(\psi), \qquad \sigma = \sigma_0 + \int \frac{d\psi}{\Phi(\psi)} \tag{16}$$

$$\psi = \frac{[P + \varphi(P)]}{R}\exp\left(-\int \frac{dP}{[P + \varphi(P)]}\right) + \int \Delta'(P)\exp\left(-\int \frac{dP}{[P + \varphi(P)]}\right)dP$$

and

$$E(P, R) = \omega(R)\,\eta\,[P / R^2\omega'(R)] \tag{17}$$

where

$$\tau = \exp\int \frac{d\xi}{[\xi - \eta(\xi)]}\,\Phi(\psi), \qquad \sigma = \sigma_0 + \int \frac{d\psi}{\psi^2\Phi(\psi)}$$

$$\xi = \frac{P}{R^2\omega'(R)}, \qquad \psi = \frac{1}{\omega(R)}\exp\int \frac{\eta'(\xi)}{[\xi - \eta(\xi)]}\,d\xi \tag{18}$$

and

$$E(P, R) = \omega(R) + \eta\,[P/R^2\omega'(R)] \tag{19}$$

where

$$\tau = (1 - \xi)\,\Phi(\psi), \quad \sigma = \sigma_0 + \int \frac{d\psi}{\Phi(\psi)}, \quad \xi = \frac{P}{R^2\omega'(R)}, \quad \psi = \omega(R) + \int \frac{\eta'(\xi)\,d\xi}{(1 - \xi)} \tag{20}$$

Here ϕ, Δ, ω, η and Φ denote arbitrary functions, each of one argument. Equations (16), (18) and (20) give the corresponding expressions for temperature and entropy.

BIBLIOGRAPHY

1. Sedov, L.I., *Metody podobiia i razmernosti v mekhanike* (*Similarity and Dimensional Methods in Mechanics*). Gostekhizdat, 1957. [English translation: Academic Press, 1960].

2. Bam-Zelikovitch, G.M., Rasprostranenie silnykh vzryvnykh voln (Propagation of strong explosion waves). Sbornik No. 4, *Teoreticheskaia gidromekhanika* pod red. L.I. Sedova (*Theoretical Hydromechanics*, No. 4. Edited by L.I. Sedov). Oborongiz, 1949.

3. Kochina, N.N. and Mel'nikova, N.S., O sil'nom tochechnom vzryve v szhimaemoi srede (On strong point-explosions in a compressible medium). *PMM* Vol. 22, No. 1, 1958.

4. Kochina, N.N., Ob osobennostiakh vblizi tsentra vzryva i o vozniknovenii dvukh udarnykh voln (On singularities near the center of explosion and on the appearance of two shock waves). *Dokl. Akad. Nauk SSSR* Vol. 126, No. 6, 1959.

5. Kochina, N.N., O svoistvakh dvizhenia nekotorykh idealnykh sred pri tochechnom vzryve (On the properties of the motion of certain ideal gases following an explosion at a point). *PMM* Vol. 24, No. 3, 1960.

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